Parametric sheet metal characterization by using Monte-Carlo and Levenberg-Marquardt: bulge test application

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Abstract
In the last past years, the sheet metal forming and more precisely the hydorforming technologies have been challenged by the improvements in the automotive and aircraft industries. In this field the numerical simulation is a useful tool to predict many phenomena present in the classical manufacturing forming process such as springback, necking, wrinkling, buckling and surface deflections. But the computational-costs for finite element simulations of general sheet metal forming processes are considerable, especially measured in time. For this reason the industrial require a high quality of the results and a good correlation with the experiment, where a good description of the sheet metal mechanical behaviour is an essential requirement.

To answer this need, several bulge tests by using circular and elliptical dies and tensile tests in three directions 0°, 45° and 90° with respect to the rolling direction have been carried out. A mixed numerical–experimental method is used to identify the plastic behavior of AISI 304 stainless steel used in many industrial application. The basic principle of the inverse method used is to compare experimentally results with the results computed by a finite element (FE) model. The Hill’s 1948 anisotropic criterion and the isotropic hardening are considered in this numerical model computed with Abaqus/Standard. The connection between numerical model and experimental results is carried out by a home made software developed within the LGP by using C++ language and called IDENTIF [1].

The difference between the experimental and the numerical responses is minimized by a combination between a Monte-Carlo algorithm based on a quasi-random research and a Levenberg-Marquardt algorithm derived from a classical Gauss-Newton. Python scripts are used to pilot the Abaqus/Standard code [3]. The bulge test with a circular die is used to identify the strain-hardening curve, the one with an elliptical die is used to identify the Lankford’s coefficients [2]. In order to valid the identified parameter, an optical method is used to measure the thickness variation along two axis of the deformed parts in order to compare them with the numerical results.

Key words: bulge test, parametric identification, plasticity, inverse methods, FEM, sheet metal, AISI304.

INTRODUCTION

The numerical simulation is the key factor to control the sheet metal forming process and reduce the development time and the final cost of products in the massive manufacturing areas such as, automotive and aerospace. Estimating material parameters for constitutive models in such a way as to describe with precision the behaviour of the material is often a common need for the engineering community [4]. Material models are becoming more and more complex, but its identification is a paramount step which precedes the design of the diverse manufacturing process of sheet metal parts (fig.1). In practice, these results of identification are exploited in input data of the numerical simulation. The last one is has been widely
introduced into the design of manufacturing products and each link of production frequently because of its high efficiency to predict several problems, in particularly the major defect present in the sheet metal forming manufacturing process are springback, necking, wrinkling, buckling and surface deflections...In order to predict this defects in the virtual manufacturing system (numerical simulation), a good description of the sheet metal mechanical behaviour is an essential requirement.

![Figure 1](image)

**Fig. 1** Diversity of sheet metal forming application in industrial manufacturing process

### 1. Bulge test

The circular hydraulic bulge test is widely used to get biaxial stress-strain curve for sheet metal forming. This test used by many authors in order to study the material formability and to proved several defects of sheet metal forming process predicted by numerical simulation software [5], [6]...

![Figure 2](image)

**Fig. 2** The geometrical bulge test condition

During this test, where the geometrical condition definite in figure 2, a circular blank is clamped at its external boundary between a die and blank holder by a drawbeads and a linearly increasing hydraulic pressure is applied in the bottom surface of the blank. Circular and elliptic dies are used. The experimental tests are performed by using a 304L stainless steel. We observe a good repetitivity of test in different pressure level, where the results are expressed in term of the applied pressure versus the pole displacement Figure 3,

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Die profile Radius</td>
<td>$r_d=6\text{mm}$</td>
</tr>
<tr>
<td>Blank radius</td>
<td>$r_b=66.5\text{mm}$</td>
</tr>
<tr>
<td>Sheet thickness ($e_0$)</td>
<td>$e_0=1\text{mm}$</td>
</tr>
<tr>
<td>Die circular diameter</td>
<td>$D_p=91\text{mm}$</td>
</tr>
<tr>
<td>Die elliptical major axis</td>
<td>$110\text{mm}$</td>
</tr>
<tr>
<td>Die elliptical minor axis</td>
<td>$74\text{mm}$</td>
</tr>
</tbody>
</table>
2. Experimental works

In this work, stainless steel material AISI 304 was selected to applied the parametric identification procedure this material. The chemical composition of the materials is given in Table 1.

<table>
<thead>
<tr>
<th>Nuance</th>
<th>C</th>
<th>Si</th>
<th>Mn</th>
<th>Cr</th>
<th>Ni</th>
<th>Mo</th>
<th>N</th>
<th>Cu</th>
<th>Fe</th>
</tr>
</thead>
<tbody>
<tr>
<td>AISI 304</td>
<td>0.05</td>
<td>&lt;0.41</td>
<td>1.14</td>
<td>18.4</td>
<td>9</td>
<td>0.193</td>
<td>0.04</td>
<td>0.348</td>
<td>Balance</td>
</tr>
</tbody>
</table>

A large experimental work is developed. In the first part of this experimental work, low rate tensile tests were carried out in the laboratory on an INSTRON tensile testing machine at a strain rate of $10^{-3}$/s. Several tensile specimens, machined by a laser cutting process from the sheet at 0°, 45° and 90° with respect to the rolling direction with standard ASTM-E8. a DIC (Digital Image Correlation) is applied in order to measured the displacement in the tensile specimen surfaces (Fig. 4 a).

![image](image1.png)

Fig. 4 (a) Tensile and (b) Bulge test apparatus

However, the standard tensile test is one of the most important engineering procedures used to characterize the mechanical behaviour of materials [10]. But in this one, the range of stable uniform strain is restricted to less than half of that sustainable under biaxial stress [11], for that, it is obviously desirable to generate the required data directly from biaxial tests. The hydraulic bulging machine equipped with an elliptical and a circular die (Fig. 4) was used to carry out several bulge test. A pattern consisting of uniformly spaced, uniformly sized circles 5.0 mm in diameter printed onto the flat sheet of steel that is to be formed, after the
bulge test, the circles change into ellipses, this last one measured to estimate the material formability study the material formability. The immediate result of a bulge test is a relation between the pressure and the height of the central point of the blank (deflexion). Then this relation is usually changed to one between stress and strain and this later used to identify the hardening law. The aim of this work is to ensure a good transformation of the output by using a combination between experimental results and numerical model.

3. Numerical and analytical model of circular bulge test

An approximate model of the mechanical test can be obtained if it is assumed that the deformed surface is spherical, that the membrane strains are equal everywhere and not just at the pole and that the thickness is uniform [7].

The thickness variation described by:

\[ e = e_0 \left(1 + \left(\frac{h}{a}\right)^2 \right)^{\frac{1}{2}} \]  

(1)

the effective strain and stress are given with the respectively (equation 2 and 3),

\[ \varepsilon = -\ln\left(\frac{e}{e_0}\right) \]  

(2)

\[ \sigma = \frac{P(a^2 + h^2)^{\frac{3}{2}}}{4he_0a^4} \]  

(3)

Circular and elliptical numerical simulation of a bulge test is overviewed in this works by abaqus software. The blank was modeled by a ‘S4R’ shell element. The boundary conditions consist of an embedding on the perimeter of the initial blank. The die modelised by riged element with respect the geometrical condition of experimental equipement (figure 5).

![Fig 5. Numerical model of the (a) circular and (b) elliptical bulge test](image)

In this parametric numerical model a isotropic hardening is considered, a fortran subroutine implemented inder abaqus in order to describe the stress-strain curve of AISI 304 metal. This relationship assumed by Swift law [7].

\[ \sigma = K(\varepsilon_0 + \varepsilon)^n \]  

(4)

where K is the strength coefficient, n is the strain-hardening exponent and \( \varepsilon_0 \) is a pre-strain.

The Hill’s quadratic yield function used in this works to introduce the anisotropic of material in forming process. The equivalent stress expressed by:

\[ \sigma^2 = (G + H) \cdot \sigma_x^2 + (F + H) \cdot \sigma_y^2 - 2 \cdot H \cdot \sigma_x \cdot \sigma_y + 2 \cdot N \cdot \sigma_{xy}^2 \]  

(5)

The Lankford’s coefficients are expressed versus the anisotropic coefficients as follows:
\[
\begin{align*}
  r_0 &= \frac{H}{G} \\
  r_{90} &= \frac{H}{F} \\
  r_{45} &= \frac{2N-(F+G)}{2(F+G)}
\end{align*}
\]

4. Parametric identification process

The aim of an parametric identification problem consists to provide a information to the numerical simulation model from knowledge of the physical process. The solution of the inverse identification is to obtain a minimum of an objective function which is defined in taking into account the numerical model of the material test and a set of experimental data.

4.1. Numerical experimental design

According to R. Fisher, the experimental design is "a technological test of maximum use of the data" [14]. In the recent years, the experimental design method used in many works, particularly [15] use this technique in the optimization of parameters forming process. In this identification process, we use a factorial design in order to decrease a number of numerical simulations. For every variable, two levels are tested, a low and a high value of the parameter to represent the limits of variation of the last one. This technique used in order to study the sensibility of the numerical response according to the material parameter variation. The material parameters it identify are \{K, n, \varepsilon_0, \Gamma_0, \Gamma_{45}, \Gamma_{90}\}.

4.2. Proposed strategy of material characterization

In this identification process, first we develop a numerical model of the circular bulge test, the results of the last one and the experimental data used in order to identify the parameter of the hardening law \{K, n, \varepsilon_0\}, in this step the anisotropy coefficient obtained by tensile test are used. Second, a three numerical model of elliptical bulge test are developed in the first model the rolling direction coincide with the major axis of die, in the second model, the major axis of ellipse is die shift of an angle 45° of the rolling direction. In the last model, the rolling direction orthogonal of the major axis. The comparison of the three model between the experimental data is carried out in same in order to determinate the optimised parameters of anisotropy \{\Gamma_0, \Gamma_{45}, \Gamma_{90}\}.

![Fig. 4. Identification Process](image-url)
4.3 The optimization methods

4.3.1 Overview of the Optimisation Techniques

For the numerical-experimental techniques application, many optimization methods are developed in order to identified the non liner physical phenomenon (behavior of material, damage, necking...).

4.3.2 Monte-Carlo method

The Monte-Carlo method (named for the Monaco casino), based on the research by using a random numbers and the statistical tools, applied in diverse filed such as mechanics, electric, nuclear physics. In this identification process, the idea is to present a factor of chance to solve a physical problem starting from a play of the parameters injected into a numerical model. The application of Monte-Carlo method required the identification of the initial parameters and the percentage of variation of each parameter. This method represent a tool to coarse research, once the criterion of Monte-Carlo is satisfied we passes to the raffinement of the solution by Levinberg-Maraquardt

4.3.3 Levenberg-Marquardt method

Levenberg-Marquardt optimization is a virtual standard in nonlinear optimization, it is a stabilized version of the Gauss-Newton method [1]. This Levenberg-Marquardt method works very well in practice and has become the standard for solving nonlinear least squares systems of equations [17].

This identification process The general procedure of the present identification problem is schematically illustrated in Fig. 5. this Identification

Fig. 5 Schema of parameter obtainisim by Monte-Carlo and Levenberg-Marquardt algorithm
In order to determine the stress–strain relationship, the aim fixed is to minimize an objective function in order to reduce the difference between the numerical and the experimental values. The objective function can be defined as in equation 5 to minimize the errors of the center point blank displacement in simulation compared with the experimental results the pole position $h$ can be recorded during the whole bulge test.

$$f = \frac{1}{m} \sum_{j=1}^{m} w_j \left( \frac{h_{\text{FEM}}[j] - h_{\exp}[j]}{r_{\exp}[j]} \right)^2$$

where

$m$ is the total number of response, $h_{\text{FEM}}$ is the vector of the simulated responses, $h_{\exp}$ is the vector of the experimental response and $w$ is the vector of the responses weights.

The minimal value of the objective function $f$ computed for the current set of parameters tested in then compared to the previous one found for the previous set. If the new one is smaller then the previous one, convergence is reached. In other case, a resumption of the drawing of lots is made without changing the starting parameters. Therefore we obtain a new distribution of sets of parameters.

5. Results and discussion

Previously, we indicated the circular bulge test of a sheet can be analysed as a thin spherical shell expanded uniformly by internal pressure. The analytical modeling, us enabled to plot the stress strain curve obtained by the equi-biaxial condition considered in the dome of deformed shape, this result are compared with the experimental tensile data and the swift law identified by the inverse method (figure ...). The result summarized in the table 2 represent the output of the parametric identification obtained by the comparison of the experimental and the numerical height of central point evolution according to the pressure value (figure...).

<table>
<thead>
<tr>
<th>variables</th>
<th>Unit</th>
<th>Experimental Value</th>
<th>Identified value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K$</td>
<td>MPa</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$n$</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$\varepsilon_0$</td>
<td>—</td>
<td>—</td>
<td>—</td>
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<tr>
<td>$r_0$</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$r_{15}$</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$r_{90}$</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

![Graph showing stress vs strain]
Fig. 5 Stress-strain curve (tensile test data, parametric identification, theoretical model)

Fig. Deflexion in the circular bulge test case

- Courbe elleptique de 0 Experimentale te identifier, et par traction data)

- Courbe elleptique de 45 Experimentale te identifier, et par traction data)

6. Thining study

The thickness variation is used an indicator of the risks undergoes by the part at the forming process. For that, it is recommended to evaluate the thickness variation during the development of a manufacturing process. Gutscher [16] found that the behavior law parameters has significant influence on the height and thickness in the bulge test. In order to evaluate the identified parameter obtained by parametric method, first, the authors using a optical method [12] to determine the experimental thickness variation in the major and minor axis of elliptical deformed shape. Second, the identified parameters injected in the numerical model, a comparison between experimental and numerical results is carried out, a good correlation is observed between this results.
7. Concluding and remarks

In this paper, a parametric identification method is applied in order to characterize a stainless steel sheet metal. This method is based on the combination of a FEM and experimental results. The cost function obtained by Monte-Carlo algorithm and refined by Levenberg-Marquardt algorithm. The numerical modelisation of the experimental bulge test carried out inside Abaqus/Standard by using an anisotropic yield criterion Hill'48 and a user hardening law is implemented (Swift law). Three steps in this identification process are investigated: First, a virtual experimental design is developed in order to study the material parameters sensitivity in the FE model. Second, the circular bulge test used in order to identify the parameters of hardening law. In the last step the helicoidal bulge test in three direction used to identify the material anisotropy.

This identification method was successfully applied to the determination of materials parameters using the experimental data. This obtained results used to evaluate the thickness variation in sheet metal forming and compared with the experimental results. This method proves to be more efficient than the classical method.

Acknowledgements

REFERENCES


