2D AND 3D NUMERICAL MODELS OF METAL CUTTING WITH
DAMAGE EFFECTS.

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Key words: Metal cutting, Damage, Finite element model, Two-dimensional, Three-dimensional, ALE.

Abstract. In this paper a two-dimensional and a three-dimensional finite element model of 
unsteady-state orthogonal cutting are presented. These models take into account dynamic 
effects, thermo-mechanical coupling, constitutive damage law and contact with friction. The 
simulations concern the study of the unsteady-state process of chip formation. The yield stress 
is taken as a function of the strain, the strain rate and the temperature in order to reflect 
realistic behaviour in metal cutting.

Unsteady-state process simulation needs a material separation criterion (chip criterion) and 
thus, many models in the literature use an arbitrary criterion based on the effective plastic 
strain, the strain energy density or the distance between nodes of parts and tool edge. The 
damage constitutive law adopted in models presented here allows defining advanced 
simulations of tool’s penetration in workpiece and chip formation. The originality is this 
damage law has been defined from tensile and torsion tests, and we applied it for machining 
process. Stresses and temperature distributions, chip formation and tool forces are shown at 
different stages of the cutting process.

Finally, we present a three-dimensional oblique model to simulate the unsteady-state process 
of chip formation. This model, using the damage law defined before, allows an advanced 
simulation close to the real cutting process.

An Arbitrary Lagrangian Eulerian formulation (ALE) is used for these simulations; these 
approach combines both the advantages of Eulerian and Lagrangian representations in a 
single description, and is exploited to reduce finite element mesh distortions.
1 INTRODUCTION

The development of a machining process requires considerable investment of time and resources. Precise knowledge about the optimal cutting parameters is essential for begin. Process features such as tool geometry and cutting speed directly influence chip morphology, cutting forces, the final product dimensionality and tool life. Computer simulation of the cutting process can reduce the number of design iterations and results in a cost saving. Considerable effort has therefore been devoted to the development of computational models of high speed machining.

Numerical models appear at the beginning of the seventh; Eulerian models have been developed since 1980\textsuperscript{1,2}. Many Lagrangian models\textsuperscript{3,4,5} have also been developed for the simulation of metal cutting. Generally, these models provide information about stresses and strain fields, shear zones, and temperature field when it’s a thermocoupled model. Some models study the friction influence on chip formation and tool forces. In 1985, Strenkowski\textsuperscript{6} has presented a thermomechanical model which predict residual stresses in the workpiece, as Shih\textsuperscript{7} in 1990. Lin\textsuperscript{8}, in 1992, has studied tool forces and compared with experiment. Sekhon et Chenot\textsuperscript{9} in 1993, have also shown tool forces and stresses distribution. Other well-known authors as Ortiz\textsuperscript{10}, Fourment\textsuperscript{11}, Usui\textsuperscript{12}, or Obikawa\textsuperscript{12} have developed unsteady-state models applied to metal cutting. The difficulty in this kind of model is to determine the method allowing elements and nodes separation and thus, chip formation. All of us have used a criterion to realise this operation. Often, this criterion of separation, generally called “chip criterion”, is based on the strain energy density. A value of a critical distance is used by Shih\textsuperscript{7}, between the tip of the cutting tool and the nodal point located immediately ahead. Obikawa, Shirakashi and Usui\textsuperscript{12} have presented a model with a double-criterion based on the value of a critical plastic strain and a geometric criterion, thus they simulate fragmented chip formation. Sekhon and Chenot\textsuperscript{9} use a plastic strain criterion. All of these criterions are generally arbitrary and are predefined on a nodal line followed by the face of cutting tool. Most of them give good results close to the real cutting behaviour. However, the use of this kind of chip criterion is arbitrary and it’s generally applied at located zone where the contact will append. A use of separation criterion applied on the whole material before machining will seem better represent the reality; this is this way that we will to develop. Better than a use of a separation criterion, a damage law, as the material behaviour law, will be used in our model.

In this paper, we present a two-dimensional and three-dimensional finite element model of unsteady-state orthogonal metal cutting. These models could simulate the formation of continuous and discontinuous chips during the process, depending of the material machining. Dynamic effects, thermomechanical coupling, constitutive damage law and contact friction are taken into account. The yield stress is taken as a function of the strain, the strain rate and the temperature. The damage constitutive law adopted here allows advanced simulations of tool’s penetration and chip formation. Stress and temperature fields, chip formation and tool forces are shown at different stages of the cutting process. Finally, we present a three dimensional oblique model to simulate the unsteady state process; it represent an extension of the model defined before.
The case of three dimensional orthogonal metal cutting has already been treated in the literature since the beginning of the nineties and notably by Lin and Lin\textsuperscript{13}. The first three dimensional oblique models have been presented by Maekawa\textsuperscript{14} in 1990, Ueda\textsuperscript{15} in 1993 and Pantalé\textsuperscript{16} in 1996. In our model we use the damage law already used in last models, which provide interesting simulations.

The continuous and fragmented chip formation induce large mesh distortions and separations. To reduce numerical problems for these simulations, an Arbitrary Lagrangian Eulerian formulation (A.L.E.) has been adopted; already used by Rakotomalala\textsuperscript{17}, Pantalé\textsuperscript{16} and Joyot\textsuperscript{18}. This approach combines both the advantages of Eulerian and Lagrangian representations in a single description, and is exploited to reduce mesh distortions.

2 FINITE ELEMENT DISCRETIZATION

The arbitrary Lagrangian-Eulerian description introduces a reference system which is not contained to remain fixed in space (Eulerian) or to move with material points in the body (Lagrangian). The reference configuration comprises a set of points (grid points) which may be identified by a set of independent co-ordinates $\xi_i$ which gives the arbitrary velocity (or grid velocity) $w_i$; in numerical treatment, the physical quantities are attached to the co-ordinates $\xi_i$.

2.1 Conservation laws in ALE description

The equations which govern the continuum in the in ALE description are the three conservation laws:

\begin{equation}
\rho + c_i \rho_j + \rho v_{ij} = 0 \quad \text{(mass equation)}
\end{equation}

\begin{equation}
\rho v_i + \rho c_j v_{ij} = \tau_{ij} + b_i \quad \text{(momentum equation)}
\end{equation}

\begin{equation}
\rho e + \rho c_i e_j = \tau_{ij} v_{(i,j)} \quad \text{(energy equation)}
\end{equation}

In these expressions, $\rho$ is the mass density, $v_i$ the material velocity, $b_i$ the body force, $\tau_{ij}$ the Cauchy stress tensor, $e$ is the internal energy, $v_{(i,j)}$ the strain rate tensor, $c_i = v_i - w_i$ is the relative velocity between the material velocity ($v_i$) and the mesh velocity ($w_i$).

2.2 Spatial discretization

In view of spatial discretization of the mass, momentum and energy equations (1), (2) and (3) by the finite element method, a classic variational form is obtained by multiplying this equation by a weighting function ($\rho, v_i, e$) over the spatial domain $w(t)$. Employing the divergence theorem, the variational forms associated with these equations are given by:
Using the Galerkin approach, one obtains the corresponding matrix equations:

\[
M^o \ddot{\rho} + N^o \rho + K^o \rho = 0 \\
M^e \dot{v} + Nv + f^\text{int} = f^\text{ext} \\
M^e e + N^e e = r
\]

\(M, M^o, M^e\) are the generalised mass matrices for the corresponding variables in (7), (8) and (9), respectively; \(N, N^o, N^e\) are the generalised convective matrices; \(K^o\) is the stiffness matrix for density; \(f^\text{int}\) is the internal force vector; \(f^\text{ext}\) is the external load vector; \(r\) is the generalised energy source vector.

Quadrilateral elements with four nodes with an integration point have been used to treat these simulations. Zero energy modes are treated by Hourglass control for the momentum equation, and the transport terms are treated by an Upwind procedure.

### 2.3 Explicit dynamic analysis

The equations of motion for the body are integrated using the explicit central difference integration rule.

\[
v^{(i+1/2)} = v^{(i-1/2)} + \frac{\Delta t^{(i+1)} + \Delta t^i}{2} \dot{v}^{(i)}
\]

where \(v\) is the velocity and \(\dot{v}\) the acceleration. The superscript \((i)\) refers to the increment number and \((i-1/2)\) and \((i+1/2)\) refer to midincrement values. The central difference integration operator is explicit in that the kinematic state can be advanced using known values of \(v^{(i-1/2)}\) and \(\dot{v}^{(i)}\) from the previous increment. The explicit integration rule is quite simple but by itself does not provide the computational efficiency associated with the explicit dynamics procedure. The key to the computational efficiency is the use of diagonal element mass matrices because the inversion of the mass matrix that is used in the computation for the accelerations at the beginning of the increment is triaxial.
\[ \dot{v}^{(i)} = M^{-1}(f^{(i)}_{\text{ext}} - f^{(i)}_{\text{int}} - f^{(i)}_{\text{c}}) \quad (11) \]

where \( M \) is the diagonal lumped mass matrix, \( f^{(i)}_{\text{ext}} \) is the external force vector, \( f^{(i)}_{\text{int}} \) is the internal force vector and \( f^{(i)}_{\text{c}} \) the convective force vector. The explicit procedure requires no iterations and no tangent stiffness matrix. Special treatment of the mean velocities is required for initial conditions, certain constraints, and presentation of results. The explicit procedure integrates through time by using many small time increments. A stable time increment is computed for each element in the mesh. A conservative estimate of the stable time increment is given by the minimum taken over all the elements. The above stability limit can be rewritten as:

\[ \Delta t = \min \left( \frac{L_c}{c_d}, \frac{1}{\sqrt{\text{det}B}} \right) \quad (12) \]

where \( L_c \) is the characteristic element dimension and \( c_d \) is the current effective, dilatational wave speed of the material. The characteristic element dimension is derived from an analytic upper bound expression for the maximum element eigenvalue. Considering the four nodes uniform strain quadrilateral element, the element dimension is:

\[ L_c = \frac{A}{\sqrt{B_{ij}B_{ji}}} \quad (13) \]

where \( B_{ij} \) is the element gradient operator.

### 3 NUMERICAL MODELS

While metal cutting is one of the most frequent operation in manufacturing today, a general predictive model of the cutting process is not yet available. The reason is that the physical phenomena associated with the process are extremely complex: friction, adiabatic shear bands, free surfaces, heating, large strains and strain rates.

The model of unsteady chip formation presented here try to take into account almost of these physical phenomena. The initial geometry and mesh are presented on Figure 1. The tool is considered to be rigid, and to moved at a constant speed \( V_c \). For this study, the cutting speed is equal to 4m/s, the depth cut \( s \) is equal to 0.5mm and the width of cut \( w \) equal to 2mm. These choices will allow experimental and numerical validation. The material used is a 42CD4 steel whose characteristics are described later. The length of workpiece is 10mm and the height is 5mm. The rigid cutting tool has a rake angle equal to 5.7° as his relief angle, and the radius of the cutting edge is equal to 0.1mm. The initial temperature of the workpiece is assumed to be 300°K. The workpiece is fixed in space and time to his base, and only move the tool. These cutting conditions are reported in Table 1 and thus we should simulate continuous chip formation.
This simulation has been modelled in a plane strain state, we used classic quadrilateral elements with four nodes. Stresses, strains and temperatures are computed at one integration point. Tool is meshed with rigid elements, and only the cutting force will be calculated on it. None other dictate conditions are imposed on tool or workpiece for chip formation.

Figure 1: Initial geometry and mesh.

4 CONSTITUTIVE AND CONTACT LAWS

4.1 Material law

The material law of original Johnson-Cook\textsuperscript{10} form is used for the simulations presented in this paper. This relationship is frequently adopted for dynamic problems with high strain rates and temperature effects. Assuming a Von-Mises type yield criterion and an isotropic strain hardening rule, the flow rule is given by:

\[ \sigma = (A + B\varepsilon^p)(1 + C\ln\dot{\varepsilon}^*))(1 - T^m) \]  \hspace{1cm} (14)

Where \( \varepsilon \) represents the equivalent plastic strain, \( \varepsilon^* = \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \) the non dimensional equivalent strain rate and \( T^* = (T - T_0)/(T_{\text{melt}} - T_0) \) is a function of temperature. \( A, B, C, n \) and \( m \) are material parameters, and according to Johnson and Cook, their values for a 42CD4 steel are given in Table 2. Johnson and Cook give values of these parameters for different materials. The data for the material constants are obtained from torsion tests (over a wide range of strain rates), static tensile tests, dynamic Hopkinson bar tensile tests and Hopkinson bar test at elevated temperatures. The model and data are evaluated by comparing computational results with data from cylinder impact tests.
4.2 Damage law

The use of a damage law was necessary to simulate unsteady-state metal cutting. As we have already said, we are not interested to include a simple arbitrary chip criterion; a damage law depending material characteristics represents a better way.

Johnson and Cook have also developed a damage law which take into account strain, strain-rate, temperature, and pressure. The originality is that this law has been defined from tensile and torsion tests; we have integrated it in our model to simulate applications such as machining process. The damage is calculated for each element and is defined by:

\[
D = \sum \frac{\Delta \varepsilon}{\varepsilon^f}
\]

(15)

where \( \Delta \varepsilon \) is the increment of equivalent plastic strain which occurs during an integration cycle, and \( \varepsilon^f \) is the equivalent strain to fracture, under the current conditions. Fracture is then allowed to occur when \( D = 1.0 \) and the concerned elements are deleted from the mesh. The general expression for the strain at fracture is given by:

\[
\varepsilon^f = \left[ D_1 + D_2 \exp D_3 \sigma^* \right] \left[ 1 + D_4 \ln \dot{\varepsilon}^* \right] \left[ 1 + D_5 T^* \right]
\]

(16)

depending on the variables (\( \sigma^* = \frac{\sigma_m}{\sigma} \), \( \dot{\varepsilon}^* \), \( T^* \)). The dimensionless pressure-stress ratio is defined as \( \sigma^* = \frac{\sigma_m}{\sigma} \) where \( \sigma_m \) is the average of the three normal stresses and \( \sigma \) is the Von-Mises equivalent stress. The dimensionless strain rate, \( \dot{\varepsilon}^* \), and homologous temperature, \( T^* \), are identical to those used in the strength model defined before (material law).

The five constants are \( D_1, D_2, D_3, D_4, D_5 \) and their values are reported Table 3. The expression in the first set of brackets follows the form presented by Hancock and Mackenzie. The expression in the second brackets represent the effect of strain-rate, and the one in the third set of brackets represent the temperature effect. The five constants are determined by tensile and torsion tests, and are different according to materials used for tests.

4.3 Contact law

While we are able to introduce any expression of tangential component of the surface traction in our model, we assume here, in the contact region between the tool and the chip, a classic Coulomb friction. The slip/Stick condition can be determined by:

\[
\text{Slip if } |T'| \geq C_f |T^n|, \text{ stick if } |T'| \leq C_f |T^n|
\]

(17)

in which \( C_f \) is the coefficient of friction; \( T^n \) and \( T' \) are, respectively, the normal and tangential components of the surface traction on the interface. A value of 0.32 is assumed for \( C_f \), determined from a specific friction test.
Tableau 1: Cutting parameters

<table>
<thead>
<tr>
<th>Material</th>
<th>42CD4</th>
</tr>
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<tbody>
<tr>
<td>( V_c )</td>
<td>4 m/s</td>
</tr>
<tr>
<td>( s )</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>( w )</td>
<td>2 mm</td>
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Tableau 2: Material law parameters

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<thead>
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<tr>
<td>( A )</td>
<td>595 MPa</td>
</tr>
<tr>
<td>( B )</td>
<td>580 MPa</td>
</tr>
<tr>
<td>( C )</td>
<td>0.023</td>
</tr>
<tr>
<td>( n )</td>
<td>0.133</td>
</tr>
<tr>
<td>( m )</td>
<td>1.03</td>
</tr>
<tr>
<td>( T_{\text{melt}} )</td>
<td>1793 K</td>
</tr>
<tr>
<td>( T_0 )</td>
<td>300 K</td>
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</table>

Tableau 3: Damage law parameters

<p>| | |</p>
<table>
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<tbody>
<tr>
<td>( D_1 )</td>
<td>1.5</td>
</tr>
<tr>
<td>( D_2 )</td>
<td>3.44</td>
</tr>
<tr>
<td>( D_3 )</td>
<td>-2.12</td>
</tr>
<tr>
<td>( D_4 )</td>
<td>0.002</td>
</tr>
<tr>
<td>( D_5 )</td>
<td>0.61</td>
</tr>
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</table>

5 NUMERICAL RESULTS AND VALIDATION

5.1 Two-dimensional model results

An explicit finite element code was used to solve this metal cutting problem. Figure 2, Figure 3 and Figure 4 show Von-Mises stress fields at different states of the simulation. Figure 5 shows the temperature corresponding at the same time that the one in Figure 4. The cutting force, during the simulation, is represented on Figure 6. Finally, we have chosen an element in the middle of the chip to obtain plastic strain evolution (Figure 7).
Figure 2: Von-Mises stresses (Pa) at time $t = 0.087$ms.

Figure 3: Von-Mises stresses (Pa) at time $t = 0.26$ms.
Figure 4: Von-Mises stresses (Pa) at time t= 1.3ms.

Figure 5: Temperature field (Kelvin) at time t=1.3ms.
Figure 6: Cutting force evolution (Newton).

Figure 7: Plastic strain evolution for an element in middle of chip.
These simulation shows tool penetration in workpiece and chip formation. In Agreement with experimentations\textsuperscript{18}, the chip is continue due to material used and coupled with cutting conditions. It has been established that the maximum value of Von-Mises stress occurs over the primary shear bands\textsuperscript{16} as it’s shows Figure 2, 3 and 4. Temperature field, Figure 5, shows the maximum value at contact between the tool rake face and the chip, due to a secondary shear band effect. It puts equally in evidence a hot temperature band on the machined surface after the pass of the tool. All these various observations agree with other model results\textsuperscript{16} and experimental observations\textsuperscript{18}. Figure 6 represents the evolution of the cutting force on tool during the process. We could see the stabilisation of it after an oscillation period due to the tool penetration and chip formation. When the chip geometry is stable, cutting force is stabilised at a value of 1800 N, agree with experimental\textsuperscript{18}. We show, for information, plastic strain evolution on one element located in middle of chip, (Figure 7). Reported in Table 4, different values are compared with Joyot\textsuperscript{18} and Pantale\textsuperscript{16} model results and with experimental and Oxley\textsuperscript{16} results.

<table>
<thead>
<tr>
<th>( \sigma_{\text{max}} ) (MPa)</th>
<th>Actual model</th>
<th>Experimental</th>
<th>Joyot model</th>
<th>Pantale model</th>
<th>Oxley</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{\text{max}} ) (( ^o )K)</td>
<td>1350</td>
<td>--</td>
<td>1500</td>
<td>1400</td>
<td>--</td>
</tr>
<tr>
<td>( F_c ) (N)</td>
<td>1800</td>
<td>1860</td>
<td>1740</td>
<td>2096</td>
<td>2328</td>
</tr>
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</table>

Tableau 4: Results comparisons.

### 5.2 Three-dimensional oblique model results

More investigations are recently been pushed for three-dimensional models of cutting metal. It seems that the first investigator, found in literature, was Maekawa in 1990\textsuperscript{14}. He has simulated unsteady-state process of oblique chip formation. Ueda and Manabe\textsuperscript{15} have also developed a three-dimensional oblique model of unsteady-state chip formation. The chip separation criterion is based on geometrical consideration of tool edge and work material. They have studied chip formation and cutting forces in function of inclination angle variation and have compared results with experiments. Original numerical and experimental studies have been presented in 1994 by Obikawa and Shirakashi\textsuperscript{22}, in which nodes separate in function of critical distance coupled with critical equivalent plastic strain. In 1996, Pantale\textsuperscript{16} present a three-dimensional oblique model for continuous chip formation formulated in A.L.E. This model was as steady-state study of metal cutting with the chip already formed, he has shown thermo-mechanical values evolutions and has putted in evidence “side-effects”. This “side effect” happens in three dimensional models which allow an extra degree for nodes, thus chip blows up at is base due to tool action. Finally, in 1999, Lin\textsuperscript{13} has presented a three-dimensional model, for continuous chip formation; the chip separation criterion used is based on the strain energy density coupled with a critical distance.

In a second time, we have realised an extension of the two-dimensional model presented before to perform a three-dimensional model of unsteady-state metal cutting. Results of
thermo-mechanical values and side-effects have also been observed, and in agreement with Pantale\textsuperscript{16} results. Finally, a three-dimensional unsteady-state oblique model has been developed and this is the one that we will present here. This model uses the same geometry and cutting parameters as the two and the three-dimensional model described before; we just give an inclination angle of $5^\circ$ to the tool. Material and damage laws are the same and this model is formulated in A.L.E. Unsteady-state of chip formation and Von-Mises stress distributions are presented on Figure 8 and 9. Temperature distribution in the chip and workpiece is shown on Figure 10. The component 1 evolution of the cutting force is presented on Figure 11. This component correspond to the projection of cutting force on direction 1.

![Figure 8: Von-Mises stresses distribution at time t=0.4ms.](image-url)
Figure 9: Von-Mises stresses distribution at time $t=1.5$ ms.

Figure 10: Temperature distribution (Kelvin) at time $t=1.5$ ms.
It has been established that the maximum value of the Von-Mises stress occurs over the primary shear bands\textsuperscript{16} as it’s shown Figure 8 and 9. Temperature field, Figure 10, shows a maximum value at contact between tool rake face and chip due to secondary shear band effect. It put equally in evidence a hot temperature band on the machined surface after the pass of the tool. Figure 11 represents the evolution of the cutting force on tool during the process. Cutting force results agree with experimental and two-dimensional models. We note that the little inclination angle does not modify the stabilised value of Ueda and Manabe. In Table 5, different values are compared with our two-dimensional model results and with experimental\textsuperscript{18} results.

<table>
<thead>
<tr>
<th>\</th>
<th>2D-model</th>
<th>Experimental</th>
<th>Actual model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{\text{max}}$ (GPa)</td>
<td>1.03</td>
<td>--</td>
<td>1.03</td>
</tr>
<tr>
<td>$T_{\text{max}}$ (Kelvin)</td>
<td>1350</td>
<td>--</td>
<td>1360</td>
</tr>
<tr>
<td>$F_{\text{c}}$ (N)</td>
<td>1800</td>
<td>1860</td>
<td>1850</td>
</tr>
</tbody>
</table>

Tableau 5: Results comparisons.
6 CONCLUSION

In this paper we have presented a two and a three-dimensional metal cutting model; the results are quite in agreement with other existing models and experiments. We have used a damage law defined from tensile and torsion tests to simulate the material damage and the chip formation. It must be emphasized that the formation of the chip results from the intrinsic behaviour of the material, then bringing a comprehensive model of what is called “machinability”. Ours actual investigations concern the simulation of milling for which the path of the tool tip is not straight one, and the simulation of saving for which the tool cannot be considered as a rigid body.

REFERENCES