Integration of a Multi-Physics Model in a Reliability-Based Design Framework: Application to Power Converters

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Abstract

Structures of power converters constitute very complex systems, the behaviour of which introduces several phenomenons (electric, thermal and mechanic). An accurate modelling of their behaviour, thus, requires a multi-physics model which makes it possible to take into account the physical phenomenon in consideration. To achieve this, a FEM code (e.g. MulPhyDo) adapted to power converters and dedicated to simulation of structure in a multi-physics context has been built in the laboratory. The aim of the present study is to include in the modelling the existence of randomness and uncertainties, particularly of interest in the case of power converters. In order to achieve this, the multi-physics FEM code is coupled with a program dedicated to structural reliability analysis, in an Oriented Object Programming context. In this way, it is possible to compute the failure reliability of the structure under consideration and the sensitivity of the reliability index to each random variable. An application example of a microelectronic package is presented.

Keywords: multi-physics model, structural reliability analysis, FORM, SORM, Importance sampling, microelectronic package.

1 Introduction

The design for reliability of structures such as microelectronic devices is a very challenging issue. In fact, microelectronics components experience a complex structural behaviour, which introduces several physical phenomenons (e.g. electric, thermal and mechanic). Moreover, their behaviour is affected by numerous uncertainties due to:

- natural variations of material properties and manufacturing process,
- unpredictable operating conditions (mission profile, environmental factors...),

and the lack of adequate model parameters for many of the materials being applied.

In order to predict the performance and the reliability of these devices more efficiently, it would be coherent to use realistic numerical simulations which, on one hand, take into account the couplings between the various existing physics, on the other hand, incorporate the uncertainties in the design parameters. In this way, it would be possible to assess the reliability of the given structure and to quantify the influence of the various input parameters on the structural reliability.

The aim of the present paper is to integrate a global reliability approach in a multi-physical finite element computation scheme. An illustration example of a solder joint low thermal fatigue reliability assessment is given.

2 Multi-physics modelling

The multi-physics modelling intends to take into account all existing physics, namely electric, thermal and mechanic. In this case, the structure is submitted to electrical loads and experiences a thermo-mechanical behaviour. The simulation procedure begins with an electrical analysis. The purpose of this analysis is to determine the field of electric potentials of the model when it is subjected to loadings and boundary conditions of electric nature (e.g. current density and potential). Once the electric analysis is finished, the heat loss by Joule effect in the circuit is calculated and introduced as loading into a thermal analysis. One can add if necessary the thermal boundary conditions (e.g. imposed temperature or heat transfer coefficient). At the end of the thermal analysis, the temperature field of the model is obtained. The temperature field then makes it possible to determine the differential expansions to which the package is subjected and, taking account of displacements and mechanical loads applied, the stress field can be computed through a mechanical analysis. The procedure for electro-thermo-mechanical analysis is shown on the Figure 1.

A finite element code devoted to multi-physics computing (e.g. MulPhyDo) has been built in the laboratory [1]. This code can perform weak couplings between electrical, thermal and mechanical physics, including floating subdomains and parallel architecture to gain computational time. Beside, it makes it possible to have different time-steps in the multi-physics integration.

3 Structural reliability methods

The purpose of structural reliability methods is to assess the probability of failure of a structure under a given criterion. These methods are based on [2]:

- The choice of a stochastic model for the set $X$ of input random variables (i.e. marginal density and correlation matrix or better, the joint density function $f_X(x)$);
• The definition of a failure scenario through the determination of a limit state function $G(\mathbf{X})$ which splits the safe domain (defined by $G(\mathbf{X}) > 0$) from the failure domain (defined by $G(\mathbf{X}) \leq 0$). The boundary defined by \{\mathbf{X} such that $G(\mathbf{X}) = 0$\} is called the limit state surface.

![Figure 1: Overview of the electro-thermo-mechanical analysis.](image)

The failure probability reads:

$$P_f = \int_{G(\mathbf{X})>0} f_X(x) \, dx$$

This probability may be evaluated using the FORM and SORM (First and Second Order Reliability Method). FORM requires initially a transformation $\mathbf{U} = T(\mathbf{X})$, where $\mathbf{U}$ is a set of uncorrelated standard normal variables. The transformed limit state function is denoted by $H(\mathbf{U})$. Then, the design point $\mathbf{u}^*$, which belongs to the limit state surface and lies closest to the origin, is obtain solving an optimization problem (e.g. $\|\mathbf{u}^*\|=\min \|\mathbf{u}\|$ under the constraint $H(\mathbf{U}) \leq 0$). The distance from the origin to the design point is the reliability index denoted by $\beta$. Finally, using a linearization of the limit state function at the design point and the circular property of the standard normal space, an analytic approximation of the failure probability is derived by: $P_f = \Phi(-\beta)$, where $\Phi$ is the standard normal integral. FORM gives also the sensitivity of the reliability index to changes in any input variable (e.g. the
direction cosines of the unit normal vector at the design point to the limit state surface. \( SORM \) approximation additionally uses the curvatures \( \kappa \) of the limit state function at the design point and the failure probability is estimated by [3]:

\[
P_f = \Phi(-\beta) \prod_{i=1}^{n-1} (1 - \beta \kappa_i)^{-1/2}
\]

(2)

where \( n \) is the number of random input variables.

To check \( FORM \) and \( SORM \) approximations, one can use simulation methods such as importance sampling, which is a Monte-Carlo simulation targeted at the design point. In this case, the failure probability is estimated by:

\[
P_f = \frac{1}{K} \sum_{i=1}^{K} \mathbb{1}[H(u^{(i)}) \leq 0] \frac{\varphi_n(u^{(i)})}{\psi(u^{(i)})}
\]

(3)

where \( K \) is the number of samples, \( \psi \) is termed “importance sampling” distribution and \( \varphi_n \) is the multinormal distribution. This method allows to obtain accurate values of the failure probability for a relatively low number of samples (e. g. 1000 samples). These three methods are illustrated in Figure 2.

![Figure 2: Failure probability approximation: (a) by \( FORM \); (b) by \( SORM \); (c) by importance sampling.](image)

In practice, the computation of the failure probability using the above methods is carried out by coupling the stochastic model (e. g. the probabilistic characterization of the input random variables) and a mechanical model (e. g. the procedure to compute the limit state function) [4]. The structural reliability methods are implemented in the software \( FERUM \) (Finite Element Reliability Using Matlab) [5] which is used in the present analysis.

4
4 Fatigue reliability-based assessment of a solder joint

4.1 Problem statement

One of the main failure modes of microelectronic packaging is the low cycle thermal fatigue of the flip-chip solder joints. In fact, semiconductor devices experience temperature variations which are caused by heat dissipation from electrical circuits. Due to the CTE mismatch between the joined components, different elongation and contractions take place in the layer. Thus, the solder joint between the layers experiences cycling shear strain.

![Figure 3: Schematic diagram of the PBGA.](image)

The structure under consideration is an element of a PBGA package made up of one chip and one substrate connected by 4x4 solder bumps (Figure 3). We want to assess the thermal fatigue reliability of the package. We assume that the package is subjected to a variable heat flux dissipated by the electrical circuits as shown on Figure 4. This variation consists on the application in a periodic sequence of 90 s of three cycles a power loss variation of 280 W followed by three other cycles of power loss variation of 140 W. We chose three cycles for each amplitude because it is generally the minimum number of thermal cycling at which a stabilized stress-strain hysteresis loop, required to use a strain energy based fatigue model, can be expected. We assume also, that both the substrate and the chip outer surface are subjected to film conduction. Their respective heat transfer coefficients are denoted $h_s$ and $h_c$ and the ambient temperature is taken equal to 70°C.
At this stage, it must be noted that the further data given to the various parameters, especially the scatter of the chosen random variables are not realistic. That why the present study should not be considered as an industrial application but only as an academic example.

4.2 Package modelling

Noting that MulPhyDo FEM code described above (section 2.) cannot yet solving nonlinear problems such as the one considered in this study, we use the finite element code ABAQUS [6] and we will only perform a thermo-mechanical fatigue analysis. For computation time reasons, a 2D finite element analysis of the package is carried out. Due to the package symmetry, only a half model needs to be made. 2D-8 nodes quad elements have been used for all the parts of the structure. Figure 5 shows a view of the FE model.

The solder joints were modelled with a steady-state creep material behaviour using the Darveaux constitutive relation [7]:

\[ \dot{\varepsilon}_{cr} = A[\sinh(\alpha \sigma)]^n \exp\left(-\frac{Q}{kT}\right) \]  

(4)
where $\dot{\varepsilon}_c$ is the steady-state creep strain rate, $A$ is a constant characteristic of the underlying micromechanism, $\alpha$ prescribes the stress level at which the power law break down, $\sigma$ is the applied stress, $n$ is the stress exponent, $Q$ is the activation energy, $k$ is Boltzmann’s constant and $T$ is the absolute temperature. For $62\text{Sn}36\text{Pb}2\text{Ag}$ solder, the material constants are: $A = 9.10^5 \text{ s}^{-1}$; $\alpha = 8.710^{-2}\text{MPa}^{-1}$; $n = 3.3$; $Q/k = 8110 \text{ K}$. We assume implicitly here that these parameters are temperature independent.

The other components material were modelled with linear elastic properties, which are supposed not to be temperature dependent. The nominal values of the elastic and thermal properties of the applied materials are given in Table 1. The parts respective materials and section thicknesses are given in Table 2.

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (kg/m³)</th>
<th>Young’s Modulus (GPa)</th>
<th>Poisson coef.</th>
<th>Thermal expansion (K⁻¹)</th>
<th>Conductivity (W.m⁻¹.K⁻¹)</th>
<th>Specific heat (J.kg⁻¹.K⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>silicon</td>
<td>2330</td>
<td>150</td>
<td>0.278</td>
<td>$3.24 \times 10^{-6}$</td>
<td>150</td>
<td>710</td>
</tr>
<tr>
<td>copper</td>
<td>8700</td>
<td>115</td>
<td>0.343</td>
<td>$1.70 \times 10^{-5}$</td>
<td>385</td>
<td>385</td>
</tr>
<tr>
<td>AlN</td>
<td>3260</td>
<td>330</td>
<td>0.25</td>
<td>$3.90 \times 10^{-6}$</td>
<td>180</td>
<td>770</td>
</tr>
<tr>
<td>62Sn36Pb2Ag</td>
<td>8600</td>
<td>20</td>
<td>0.35</td>
<td>$2.4 \times 10^{-5}$</td>
<td>38</td>
<td>173</td>
</tr>
</tbody>
</table>

Table 1: Nominal values of elastic and thermal material properties.
<table>
<thead>
<tr>
<th>Part</th>
<th>Material</th>
<th>Section thickness (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>substrate</td>
<td>AlN</td>
<td>0.02</td>
</tr>
<tr>
<td>chip</td>
<td>silicon</td>
<td>0.0135</td>
</tr>
<tr>
<td>intermetallic layer</td>
<td>copper</td>
<td>0.0135</td>
</tr>
<tr>
<td>solder ball</td>
<td>copper</td>
<td>0.002</td>
</tr>
<tr>
<td>solder joint</td>
<td>62Sn36Pb2Ag</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Table 2: Package parts material and section thickness.

4.3 Probabilistic model for input variables

In this example, we consider six random parameters, namely, both the substrate and the chip outer surfaces heat transfer coefficients, the thermal conductivity and the Young’s modulus of the solder joint material, and both the substrate and the chip coefficients of thermal expansion (CTE). The other parameters, in particular the geometrical dimensions, are supposed to be deterministic. Note that, no uncertainty is introduced in the modelling parameters of the solder joint creep behaviour. The statistical properties of the random variables are given in Table 3.

<table>
<thead>
<tr>
<th>Variable description</th>
<th>Distribution</th>
<th>Mean</th>
<th>Coef. of var.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat transfer coef., $h_s$</td>
<td>log-normal</td>
<td>20000</td>
<td>0.1</td>
</tr>
<tr>
<td>Heat transfer coef., $h_c$</td>
<td>log-normal</td>
<td>20000</td>
<td>0.1</td>
</tr>
<tr>
<td>Solder joint thermal conductivity</td>
<td>log-normal</td>
<td>38</td>
<td>0.1</td>
</tr>
<tr>
<td>Solder joint Young’s modulus</td>
<td>log-normal</td>
<td>20 $10^{-6}$</td>
<td>0.1</td>
</tr>
<tr>
<td>Substrate CTE</td>
<td>log-normal</td>
<td>3.90 $10^{-6}$</td>
<td>0.1</td>
</tr>
<tr>
<td>Chip CTE</td>
<td>log-normal</td>
<td>3.24 $10^{-6}$</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 3: Statistical properties of the random variables.

4.4 Low-cycles thermal fatigue modelling

The Darveaux energy-based fatigue model is used for determining the crack initiation in the structure [8]:

$$N = K_1 \Delta W_{ave}^{K_2}, \quad \Delta W_{ave} = \frac{\sum \Delta W_e \cdot V_e}{\sum V_e} \quad (5)$$

where, $\Delta W_e$ is the dissipated energy for one element, which is the area contained in the stress-strain hysteresis loop, $\Delta V_e$ is the associated element volume and $\Delta W_{ave}$ is
an average dissipated energy computed by weighting the energy for each element with its volume and then normalize with the total volume. This fatigue model makes it possible to reduce the effect of geometric singularities. The respective values of the model parameters are \( K_2 = -1.64 \) and \( K_1 = 13.776 \text{ cyc/MPa}^{K_2} \).

In this example, the crack is assumed to initiate in the periferic solder joint, connecting the chip to the ball solder via an intermetallic layer, since this is where the largest dissipated energy occurs (Figure 5). The linear Miner’s rule is used to cumulate fatigue damage caused by both cycling heat flux variations of the sequence. Thus, denoting \( N_1 \) and \( N_2 \) the respective numbers of allowable cycles for both variations of the heat flux on the given sequence (Figure 4), the fatigue damage caused by one sequence reads:

\[
D = \frac{1}{N_1} + \frac{1}{N_2}
\]  

(6)

Then the life-time of the structure is given by: \( N = 1/D \) and is expressed in terms of number of sequences.

### 4.5 Reliability analysis

Now, let us compute the thermal fatigue failure probability of the structure. The reliability design criterion is defined as follows: given a service life-time \( N_0 \), the failure occurs when the package fails before reaching the given service life-time \( N_0 \). The limit state function is therefore express by:

\[
G = N - N_0 \text{ or equivalently by } G = \ln(N) - \ln(N_0)
\]  

(7)

The second formulation of the limit state function is preferred since the applied logarithm smooth the limit state surface. In this study, the service life-time is given the value of 500 sequences. The failure probability is then computed by FORM and SORM, importance sampling results are not available yet, due to CPU reasons. The results are reported in Table 4. The respective sensitivity factors of the random variables in the structural reliability are shown in the diagram in the Figure 6.

<table>
<thead>
<tr>
<th>Reliability method</th>
<th>Reliability index</th>
<th>Failure probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>FORM</td>
<td>1.88</td>
<td>3.005 ( 10^{-2} )</td>
</tr>
<tr>
<td>SORM</td>
<td>2.41</td>
<td>7.98 ( 10^{-3} )</td>
</tr>
</tbody>
</table>

Table 4: Reliability index and failure probability obtained by FORM and SORM analysis
The relatively large discrepancy between the reliability index respective values by FORM and SORM, shows that the limit state surface would have strong curvatures. Concerning the sensitivity factors, their signs are coherent with the expected effect on the reliability of any increase of the associated random variable. In fact, any positive variation of the heat transfer coefficients improve the thermal resistance of the structure, which agree with the positive sign of their sensitivity factor. In other hand, any positive variation of the solder thermal conductivity or Young’s modulus, whose sensitivity factors are negatives, increase the stress magnitude in the solder joint, and reduce logically its reliability. Surprisingly, the CTE of both substrate and chip have a negligible importance. This may be due to the fact that they were both taken as random variables. It would be instructive to consider only the discrepancy between them as a single random variable, which was not done in the present study. However, it should be emphasized that, since the obtained results have not been yet checked by a simulation method no definitive conclusion can be made. In fact, the limit state surface can have a complex form with several optimums due to the presence of strong non-linearity of the problem (e. g. creep behaviour of the solder material).

5 Conclusion

In the present work, global probabilistic analysis integration in a multi-physics model is presented. It aims at incorporating both the coupling between various physics in consideration and the existence of uncertainties in the parameters of the simulation. This approach can be of particular interest in the microelectronic industry whose component experience hardly predictable operating conditions and whose material model parameters present large scatter. In fact, it proposes a coherent analysis of such structures and its results (e. g. failure reliability, sensitivity factors) make it possible to better understand and quantify the various factors influencing the structural response through accurate numerical modelling. The academic illustration example presented in this paper prior to treat an industrial application, intends to show that it is possible to assess low cycle thermal fatigue reliability of solder joints.
Figure 6: Sensitivity factors.

References