A THREE-DIMENSIONAL NUMERICAL MODEL OF ORTHOGONAL AND OBLIQUE METAL CUTTING PROCESSES

Olivier PANTALE
Laboratoire Génie de Production C.M.A.O
E.N.I.T Av d’Azereix BP1629
65016 Tarbes

Maurice TOURATIER
LM²S UA CNRS 1776, ENSAM
Univ. P&M Curie 151 bd de l’hôpital
75013 Paris

Roger RAKOTOMALALA
Laboratoire Génie de Production C.M.A.O
E.N.I.T Av d’Azereix BP1629
65016 Tarbes

Nawel HAKEM
Direction de la Recherche RENAULT
860 quai Stalingrad
92109 Boulogne Billancourt

Abstract
An Arbitrary Lagrangian Eulerian (ALE) approach is used to model continuous tridimensional orthogonal and oblique steady state metal cutting processes. The use of an ALE formulation enables a continuous update of the free and contact surfaces of the model, without needing iterative computations as in a purely Eulerian formulation. Hence, the use of an appropriate grid movement control algorithm may enable accurate computations without remeshing as in a Lagrangian formulation. The thermomechanical model includes the effects of elasticity, strain rate, temperature and friction between the chip and the tool. A thermal-viscoplastic constitutive equation associated to flow law of Johnson-Cook is adopted for the workpiece. The use of an ALE formulation enables to model the tool-chip contact without needing a separation criteria in the front edge of the tool. A classical Coulomb friction law associated with heat generation, and heat transfer is used to model the tool-chip contact. The model is suitable to predict thermomechanical quantities, the chip geometry and the cutting forces from a set of cutting data, material and contact parameters. Cutting experiments and numerical simulations were performed on a 42CD4 steel and comparisons show a reasonable level of agreement.

1 INTRODUCTION

During a metal cutting process, the material is subjected to high strains, high strain rates and high temperatures. The interactions between the tool and the workpiece are also very complex. All these complexities lead to several nonlinear aspects in governing equations. To simplify the modelling, many analytical and numerical investigations on metal cutting processes are relative to the orthogonal cutting in shear plane problem.

Most of the existing numerical models are based on Lagrangian approach. Strenkowski and Carol [13] use an updated Lagrangian method, with a finite element solution. Their model is operated to simulate the transient chip formation and the steady state problem. However, the severe distortions of the finite element mesh affect the numerical solution of the problem; in addition, a geometrical separation criterion must be introduced to separate the chip from the workpiece. Shih and Yang [12] adopt a similar approach for the development of a thermomechanical model. Sekhon and Chenot [9], by contrast have used a mesh adaptivity enable an arbitrary surface of separation. More recently, Marusich and al. [8] have proposed a Lagrangian finite element model with continuous remeshing and material fracture simulation.

Another way to model the cutting process is to use a purely Eulerian approach. Strenkowski and Moon [14] have used such an approach and an iterative scheme is introduced.
to obtain the steady state geometry. Using an Eulerian approach in such a case gives the opportunity to avoid the severe mesh distortions.

During the past few years, the Arbitrary Lagrangian Eulerian (ALE) formulation method has been used by several authors [2][4][5], in order to overcome problems encountered while using a purely Lagrangian or Eulerian method. The use of such an approach combine both the advantages of the two classical representations in a single description which can be considered as an automatic and continuous re zoning method. In [11] the method has been applied to a bidimensional simulation of orthogonal cutting processes.

In this paper, we only study the steady state problem and the ALE method is applied to update free surfaces and the tool-chip contact surface. It means that the model is equivalent to an Eulerian one when the steady state geometry is reached. The cases of orthogonal and oblique cutting in a 3D situation are simulated to illustrate the strategy.

2 GOVERNING ALE EQUATIONS

The ALE formulation is an extension of both classical Eulerian and Lagrangian descriptions [6]. In this approach, and aiming a numerical treatment, grid points may have an arbitrary motion defined by a set of relations \( x_i = \Psi_i(x_j,t) \), and the material motion is given by \( x_i = x_i(X_j,t) \). In these relations, \( x_i, x_j \) \( X_i \) and \( X_j \) denote respectively the spatial, Lagrangian and reference grid coordinates. In an ALE numerical computation, the physical quantities are evaluated at grid points \( x_i \) which occupy geometrical points \( x_i \) at time \( t \). Hence, the material derivatives occurring in conservation and constitutive laws must be expressed by taking into account both mesh evolution and material motion. From the grid motion and material motion defined above, one can define respectively the grid velocity \( \vec{v}_i \) and the material velocity \( v_i \) by:

\[
\vec{v}_i = \frac{\partial x_i}{\partial t} \bigg|_{x_i = \text{cte}} ; v_i = \frac{\partial x_i}{\partial t} \bigg|_{x_i = \text{cte}}
\]

Then, to obtain the ALE expressions of conservation laws, it must be used the fundamental relation between the mixed derivative and the material derivative defined respectively by:

\[
\left( \frac{\partial}{\partial t} \right)_x = \left. \frac{\partial}{\partial t} \right|_{x = \text{cte}} \quad \text{and} \quad \left( \frac{\partial}{\partial t} \right)_x = \left. \frac{\partial}{\partial t} \right|_{x = \text{cte}}
\]

The term \( \left( \frac{\partial}{\partial t} \right)_x \) must be interpreted as the "time" variation of a physical quantity for a given grid point.

The fundamental relation, established by the means of the motion laws and the Leibnitz rule is given by [6]:

\[
\frac{\partial}{\partial t} \left( \vec{v} \right) + \vec{c} \cdot \nabla \left( \vec{v} \right) = \nabla \cdot \left( \rho \vec{f} \right) + \rho \text{div} \vec{v}
\]

where \( c_i = \frac{\partial}{\partial t} \) is the relative velocity between material and grid points, and \( \nabla \) is the gradient operator.

The use of the relation (3) leads the local ALE formulation of the conservation laws:

\[
\rho \frac{\partial}{\partial t} \vec{v} + \rho \text{div} \vec{v} = 0 \quad \text{(mass)}
\]

\[
\rho \left( \frac{\partial}{\partial t} + \vec{c} \cdot \nabla \right) \vec{v} = \vec{f} + \text{div} \sigma \quad \text{(momentum)}
\]

\[
\rho \left( \frac{\partial}{\partial t} + \vec{c} \cdot \nabla \right) e = \sigma : D + \text{div} \vec{q} \quad \text{(energy)}
\]

In these equations, \( \rho \) is the mass density, \( \nabla \) the material velocity, \( \vec{f} \) the body force, \( e \) the specific internal energy, \( \vec{q} \) the heat flux rate, \( \sigma \) the Cauchy stress tensor and \( D \) is the strain rate tensor.

In the same way, when the constitutive relationship is given by a rate form, the standard constitutive form must be rewritten by using the relation (3). In the context of cutting modelling, a thermal-elastoplastic rule is adopted for the workpiece in our study. The Cauchy stress tensor \( \sigma \) is related to the strain rate tensor \( D \) as follow:

\[
\nabla \sigma = C : D
\]

where \( \nabla \sigma = \sigma + \Omega \sigma - \sigma \Omega \) represents the Jaumann derivative of the tensor \( \sigma \), \( C \) is the constitutive tensor and \( \Omega \) is the spin tensor. Hence, from the relation (3) the ALE form of the constitutive law is obtained and given by:

\[
\frac{\partial}{\partial t} \sigma + \vec{c} \cdot \nabla \sigma = C : D + \text{div} \sigma - \sigma \Omega
\]

The boundary conditions associated to these equations are given on figure 3.

3 DISCRETIZATION AND EXPLICIT INTEGRATION

A finite element method (FEM) is adopted for the discretization of the momentum equation (5). The associate
weak form is deduced by premultiplying the equation (5) by a weighting function \( v_i^* \) over the spatial domain \( \omega(t) \) with the boundary \( \Gamma(t) \). Using the divergence theorem to include the force \( F \) on \( \Gamma(t) \), and a classical Galerkin approach, finally one obtain the corresponding discretized matrix equation:

\[ M^* \dot{v} + F^{\text{int}} + F^{\text{ext}} + F^{\text{hr}} = 0 \]

where \( M^* \) is the mass matrix, \( F^{\text{int}} \) is the momentum transport force vector, \( F^{\text{int}} \) the internal stress force vector and \( F^{\text{ext}} \) the external force vector including friction. A three-dimensional model is constructed using 8 nodes hexahedron finite elements with one Gauss point (reduced integration) in order to reduce computing cost. Hence the corresponding zero energy modes must be controlled. Koslov and Frazier method [10] is adopted to avoid zero energy modes, and \( F^{\text{hr}} \) represents the Hourglass resistant force in the momentum equation.

Mass and energy equations are discretized using a finite volume method (FVM). In order to stabilize the conservation equations, it is often appropriate to introduce a selective ‘upwind’ into the discrete form of the transport terms. Hence, a numerical diffusion term is added to the discrete equation, and stability may be obtained using a simple explicit integration scheme. According to the control volume presented in figure 1,

\[ \int_{\omega(t)} (\rho \dot{e}) \, dw(t) + \int_{\Gamma(t)} \rho \, \overrightarrow{e} \cdot \overrightarrow{n} \, d\Gamma(t) = \]

\[ \int_{\omega(t)} \sigma \cdot D \, dw(t) + \int_{\Gamma(t)} \overrightarrow{q} \cdot \overrightarrow{n} \, d\Gamma(t) \] \hspace{1cm} (10)

lead to the corresponding discrete equations given by:

\[ V \, \dot{\rho}^o + \sum_{i=n,s,e,w,t,b} A^i \, \dot{\rho}^i \overrightarrow{c} \cdot \overrightarrow{n}^i = 0 \] \hspace{1cm} (11)

\[ V \, (\rho^o \, \dot{e}^o) + \sum_{i=n,s,e,w,t,b} A^i \, \rho^i \, \dot{e}^i \overrightarrow{c} \cdot \overrightarrow{n}^i = \]

\[ V(\sigma : D)^o + \sum_{i=n,s,e,w,t,b} A^i \, q^i \overrightarrow{n}^i \] \hspace{1cm} (12)

In these last equations \( V \) represents the volume of the cell, \( A^i \) and \( \overrightarrow{n}^i \) are the area and external normal vector of the lateral face \( i \), \( \rho^o \) and \( e^o \) represent respectively quantities \( \rho \), \( e \) computed at point \( p \) (middle of the cell).

A total compatibility between FEM and FVM is ensured setting control volumes and identical elements.

For the energy equation, Fourier’s law is adopted for the conduction term and specific internal energy \( e \) is linked to temperature \( T \) using classical relation \( de = C_p \, dT \), where \( C_p \) is the specific heat coefficient.

Concerning time integration, an explicit central difference scheme of third order is adopted. The time increment \( \Delta t \) is subjected to the classic Courant stability condition. An explicit integration scheme is adopted to solve the numerical problem. The use of an ALE method also requires an automatic mesh displacement prescription algorithm. The algorithm proposed by Giuliani [3], which is used in the model, is based on a geometrical criterion so as to minimize mesh distortions. The components of grid velocity at a typical ALE node \( I \) are computed at each step of the time integration procedure by the following relationship:

\[ \overrightarrow{v}_I^{t+\Delta t} = \frac{1}{N} \sum_{j=1}^{N} \overrightarrow{v}_j^{t-\Delta t} + \frac{\alpha}{N^2 \Delta t} \sum_{j=1}^{N} \sum_{j=1}^{N} \frac{\overrightarrow{u}_I^j - \overrightarrow{u}_I^{t+\Delta t}}{L_{Ij}} \]

where \( N \) indicates the number of nodes connected to node \( I \) via sides and diagonals; \( L_{Ij} \) is the current distance between the node \( I \) and the connected node \( J \); \( \overrightarrow{u} \) represents the total node displacement and \( \alpha \) is an upwind coefficient.

Figure 1: Description of a finite volume cell.

The mass and energy equations associated to the following conservative forms (9) and (10)

\[ \int_{\omega(t)} \ddot{\rho} \, dw(t) + \int_{\Gamma(t)} \rho \, \overrightarrow{c} \cdot \overrightarrow{n} \, d\Gamma(t) = 0 \] \hspace{1cm} (9)
THE ALE CUTTING MODEL

The model presented in this paper is used to simulate steady state cutting processes. We present a three-dimensional model of the orthogonal metal cutting, remembering the definition of the orthogonal cutting is cutting and advancing velocities are both orthogonal to the edge of the tool. The model presented here is a numerical simulation of a turning process as shown on figure 2.

Figure 2: Turning process associated to the simulation

Figure 3 shows the model used for the simulation of the cutting process. In addition to the ALE nodes of the workpiece, a typical finite element mesh generally contains purely Eulerian or purely Lagrangian nodes. By definition, an Eulerian node has a zero grid velocity \( \mathbf{\bar{v}} = 0 \) while a Lagrangian node moves with the corresponding material node \( \mathbf{\bar{v}} = \mathbf{v} \). For the description of the free surfaces of the workpiece, the use of an ALE description enables the user to create nodes which are simultaneously purely Lagrangian in normal direction and Eulerian in tangential direction. This allows a continuous update of the free surface location until the free surface condition is obtained (velocity normal component equal to zero).

Neither geometry of the chip nor the contact length are known at the beginning of the calculus. So we introduced in our model an arbitrary initial geometry which is updated during the computation.

A similar description has been used to model the tool-chip contact in the model, taking into account of heat generation due to friction forces and heat transfer between the chip and the tool. In this contact region, a classic Coulomb

Figure 3: Boundary conditions of the model used for the simulation friction is assumed. The stick/slip condition is given by:

\[
\begin{align*}
\text{stick} & \quad |T_t| \geq C_f |T_n| \\
\text{slip} & \quad |T_t| < C_f |T_n|
\end{align*}
\]

where \( T_n \) and \( T_t \) represent respectively the normal and the tangential components of the surface traction at the interface.

The heat generation in the slipping contact surface is given by: \( dQ_{frc} = |T_t| \cdot |V_c| \, dt \) where \( V_c \) is the tangential slip velocity. In the model, it is assumed that the frictional heat source is distributed fairly between the workpiece and the tool.

The material law of Johnson-Cook [7], frequently adopted for the numerical simulations of dynamic problems with high strain rates and high temperatures is adopted for the workpiece, assuming a Von-Mises equivalent stress and an isotropic strain hardening rule. The flow rule is then given by:

\[
\sigma_{eq} = (A + B \bar{e}^m)(1 + C \ln \bar{e}^*)(1 - T^m) = 0
\]

where \( A, B, C, n, m \) are material parameters and \( \bar{e}, \bar{e}^*, T^* \) are respectively the equivalent strain, the adimensional equivalent strain rate and the adimensional temperature.

The material parameters for the 42CD4 steel used in our computations and given by reference [1], are reported in table 1. The friction coefficient \( C_f = 0.32 \) has been obtained from an experimental apparatus by applying a normal force on a tool in contact with the rotating workpiece, and measuring the corresponding tangential forces.
5 NUMERICAL RESULTS

The ALE explicit approach presented above has been adopted to solve the metal-cutting problem. Machining conditions are given in table 2. We introduced a beveled edge as an approximation of the real rounded edge between rake and flank faces of the tool.

5.1 Orthogonal model

We have introduced a symetric plane to reduce the size of the model.

5.1.1 Numerical simulation

Both global and local results are available from this model. Cutting and advancing forces are \( F_c = 2.208kN \), \( F_a = 0.690kN \). Two contourplots are reported on the figures 4 and 5.

Figure 4 shows the Von-Mises stress contourplot on the side of the workpiece. The side of the workpiece is subjected to plane stress state, and the middle (the symmetric plane) to plane strain state.

Figure 5 shows a temperatures contourplot on the tool rake face. Temperatures range from 300° to 1012°. The measured distance between the hottest point and the edge is about 50% of the contact length, and agrees with experiments.

There is also a lateral dilatation of about 14% in the direction of the tool edge.
analysed surface 2.412 x 4.020 mm
depth of crater 14.7 μm
width of crater 850 μm
width of the edge 225 μm
palping conditions and results

| Table 3 |

Comparison between numerical and experimental results, given in table 4, show a good agreement level.

<table>
<thead>
<tr>
<th>experimental</th>
<th>numerical</th>
</tr>
</thead>
<tbody>
<tr>
<td>contact length</td>
<td>1.075 mm</td>
</tr>
<tr>
<td>hottest point distance</td>
<td>0.650 mm</td>
</tr>
</tbody>
</table>

5.2 Oblique model

The second model presented in this paper is about oblique cutting. To build this model, we have introduced a 15° angle between the normal of the edge and the cutting direction. Others cutting parameters are the same than those presented in table 2.

Cutting, advancing and lateral forces are $F_a = 1.811kN$, $F_a = 0.381kN$, $F_a = 0.539kN$.

Results relative to this model are reported on figure 8 (equivalent Von-Mises stress contourplot) and figure 9 (temperature contourplot).

Those results show that introducing an angle in the tool direction has an effect on the steady-state solution of the problem. First we can say that the oblique cutting model is a real three-dimensional one (no symmetric plane) and cannot be reduced to a bi-dimensional shear plane problem.

The introduction of a 15° angle has an influence on the exhaust direction of the chip. In an orthogonal model this direction in the $V_c, V_a$ plane. In the model presented here, this direction makes an angle of about 45°, as shown on figure 8 and 9.

6 Conclusion

An ALE formulation and a solution procedure for a three-dimensional steady-state cutting process have been developed and presented. Different observations deduced from numerical results have been compared with available experimental results and give a good agreement level.

1 distance between the tool edge and the hottest point on the rake face of the tool.
onal and oblique cutting processes presented in this paper are just particular cases. Further extensions of this work may include a general three-dimensional model taking into account of two cutting edges.

References


