A METHODOLOGY FOR THE IDENTIFICATION OF CONSTITUTIVE AND CONTACT LAWS OF METALLIC MATERIALS UNDER HIGH STRAIN RATES

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ABSTRACT

This paper deals with a methodology for the identification of constitutive and contact laws for metallic materials under dynamic loading at high strain rates. The work is based on a FE numerical simulation of the process and a direct Taylor impact test. The first part presents the isotropic/kinematic integration algorithm used in our model. In the second part, an iterative and generic procedure for the identification of the Johnson-Cook flow law parameters is presented, with an application to the 4340 steel.

KEYWORDS

Taylor test, constitutive law, Johnson-Cook law, identification procedure, isotropic/kinematic hardening

INTRODUCTION

Numerical simulations of forming processes and of structures under impact require constitutive, contact and rupture laws incorporable to numerical models at a scale the dimension of which is about the size of the finest finite element permitted by economic considerations . This is an industrial need, even if the physical meaning of certain parameters is not always clear at this scale. It's a typical research goal to (relie) these parameters with other material properties from microscopic considerations, then reducing their number and (amelioring) the quality of models. The identification of such laws need the use of three main tools: a specific experimental test, a FE code and an algorithm of minimization of the gap between experimental and computed responses. In the field of dynamic applications, such as High Speed Machining, the well-known Hopkinson bar tests cannot cover simultaneously high strain ranges and high strain rates concerned; so, we consider impact tests and a "post-mortem" procedure, based on measurements on the specimen after impact of chosen features (the final length and the mushroom diameter in a Taylor test [2], for instance) and on a set of calculus.

Values obtained for the parameters depend on the way of managing the three tools, and particularly their "consistence", i.e. the equivalence of noise in experimentation and in the models, including the code itself. As an example, consider the classic form of the original Johnson-Cook law [3]

$$\sigma = (A + B\overline{\varepsilon}^n)\left(1 + C\ln\frac{\dot{\overline{\varepsilon}}}{\dot{\overline{\varepsilon}}_0}\right)\left(1 + T^{*^m}\right) \tag{1}$$

in which σ, $\overline{\varepsilon}$ and $\dot{\overline{\varepsilon}}$ are respectively an equivalent stress, plastic strain and rate strain, $\dot{\overline{\varepsilon}}_0$ a quasi-static strain rate and T^* an homogeneous temperature, A, B, C, n and m being the parameters to identify.

From the database produced by the authors, and with the use of an explicit code (Abaqus Explicit), the values of A, B, C, n and m are respectively: $A = 829MPa$, $B = 483MPa$, $C = 0.0138$, $n = 0.2523$ and $m = 1.0764$ when an isotropic hardening is considered in the code (the default version), and: $A = 644MPa$, $B = 500MPa$, $C = 0.2851$, $n = 0.0142$ and $m = 1.0228$ when a kinematic hardening is assumed, for a similar quality of the identification (about 3% between experimental and computed responses), using the technique described below. In this case, the choice of the code is of the same importance as the experimental noise.

This paper concerns the taking into account of the hardening nature, by introducing a new parameter, named β, varying from 0 (kinematic hardening) to 1 (isotropic hardening). We first present the implementation of the modified plastic radial scheme, and next an identification procedure based on the work of Allen and Rule [1].

ISOTROPIC/KINEMATIC HARDENING ALGORITHM

We introduce here a very brief presentation of the J_2 plasticity theory and time integration. Many presentations of this theory are available in literature, full developments may be found in Simo [5] for example.

A brief J_2 plasticity theory

A choice of internal plastic variables which is typically of metal plasticity is $\mathbf{q} = \{\sigma_v,\, \alpha\}$ where σ_v is the equivalent plastic stress that defines the isotropic hardening and α defines the center of the Huber von-Mises yield surface in the stress deviator space (the back-stress) for a kinematic hardening. With the use of a co-rotational description, the hypoelastic constitutive relation may be written as:

$$\dot{\boldsymbol{\sigma}} = \left(K\mathbf{1}\otimes\mathbf{1} + 2G\left(\mathbf{I} - \frac{1}{3}\mathbf{1}\otimes\mathbf{1}\right)\right) : \left(\mathbf{D} - \mathbf{D}^p\right) = \mathbf{H} : \left(\mathbf{D} - \mathbf{D}^p\right) \tag{2}$$

where K is the bulk modulus, G is one of the Lamé coefficients, $\dot{\boldsymbol{\sigma}}$ is the time derivative of the Cauchy stress tensor, $\mathbf{D}$ is the total strain rate tensor, $\mathbf{D}^p$ is the plastic strain rate tensor, $\mathbf{1}$ is the Kronecker tensor and $\mathbf{I}$ is the fourth-order unit tensor. All quantities here and after are assumed to be co-rotational ones.

We also assume an additive decomposition of the strain rate tensor $\mathbf{D} = \mathbf{D}^e + \mathbf{D}^p$ and the use of the Huber von-Mises yield criteria:

$$f(\sigma,\, \alpha,\, \sigma_v) = \sqrt{\frac{3}{2}\phi : \phi} - \sigma_v = 0 \tag{3}$$

with $\phi = \mathbf{s} - \alpha$, where $\mathbf{s}$ is the deviatoric part of the Cauchy stress tensor. Introducing the normality condition, in a plastic increment, one may define the plastic strain rate tensor with the following relationship:

$$\mathbf{D}^p = \Lambda \mathbf{n} \tag{4}$$

where Λ is the consistency parameter given with the consistency condition $\dot{f}(\sigma, \alpha, \sigma_v) = 0$ and $\mathbf{n} = \frac{\phi}{\sqrt{\phi:\phi}}$ represent the external unit normal to the yield surface.

During an elastic deformation $f < 0$ and $\Lambda = 0$ and during a plastic loading $f = 0$ and $\Lambda \neq 0$, therefore, the elasto-plastic loading may satisfy the Kuhn-Tucker conditions $\Lambda f = 0$; $\Lambda \geq 0$; $f \leq 0$. The yield condition flow rule and hardening law are given by:

$$\dot{\sigma}_v = \beta \sqrt{\frac{2}{3}} h_i \Lambda \quad \text{and} \quad \dot{\alpha} = (1 - \beta) \frac{2}{3} h_k \Lambda \mathbf{n} \tag{5}$$

where h_i is the isotropic hardening coefficient, h_k is the kinematic hardening coefficient and $\beta \in [0, 1]$ is a scalar variable used to give a balance between the isotropic hardening ($\beta = 1$) and the kinematic hardening ($\beta = 0$).

Time integration

For the time integration of the above equations, we used an explicit integration scheme. The simplicity of the von-Mises yield condition (an hypersphere in stress deviator space) enables one to obtain essentially a closed-form solution of the previous equations using a so-called radial return method. This method is based on an additive decomposition of the elasto-plastic operator into an elastic prediction and a plastic correction. Concerning the stresses, the elastic predictor $\boldsymbol{\sigma}^{tr}$, so-called trial stress, may be obtained after the integration of $\dot{\boldsymbol{\sigma}} = \mathbf{H} : \mathbf{D}$ with the initial condition $\boldsymbol{\sigma}(t_0) = \boldsymbol{\sigma}_0$. If the elastic predictor is out of the yield surface, the Kuhn-Tucker conditions are not satisfied and $\boldsymbol{\sigma}^{tr}$ is used as an initial condition for the plastic corrector obtained after integrating the second part of equation (2). Therefore, we must compute the plastic corrector defined by:

$$\dot{\boldsymbol{\sigma}}^{corr} = -\mathbf{H} : \mathbf{D}^p = -2G\Lambda \mathbf{n} \tag{6}$$

and the new yield stress and back-stress given after the integration of equation (5). So the final stress (at the end of the elasto-plastic load) is given by $\boldsymbol{\sigma}_1 = \boldsymbol{\sigma}^{tr} - 2G\Gamma \mathbf{n}$ with $\Gamma = \int_{t_0}^{t_1} \Lambda dt$. The Γ parameter is calculated with respect of the condition $f(\Gamma) = 0$ at time $t = t_1$ using a local Newton-Raphson iterative procedure:

$$f(\Gamma) = \left(\mathbf{s}^{tr} - 2G\Gamma \mathbf{n} - \alpha_1(\Gamma)\right) : \left(\mathbf{s}^{tr} - 2G\Gamma \mathbf{n} - \alpha_1(\Gamma)\right) - \frac{2}{3}\sigma_v^2(\Gamma) = 0 \tag{7}$$

with $\alpha_1(\Gamma) = \alpha^{tr} + \frac{2}{3} h_k \Gamma \mathbf{n}$. When Γ is obtained, one may update the equivalent plastic strain using $\bar{\varepsilon}_1^p = \bar{\varepsilon}_0^p + \sqrt{\frac{2}{3}}\Gamma$.

In this algorithm, for large time steps, $\mathbf{s}^{tr}$ may be far outside yield surface leading to numerical inconsistencies for an implicit integration. Our model is based on a explicit integration scheme, so the elastic predictor is very close to the yield surface and the solution of equation (7) is usually obtained at the first iteration. This theory has been used to implement a isotropic/kinematic hardening constitutive law in a Abaqus/Explicit VUMAT Fortran subroutine.

An isotropic/kinematic hardening example

To illustrate the influence of the isotropic/kinematic coefficient β, we used our constitutive model to compute for a Taylor impact test the final length $L_f = f(\beta)$ and final mushroom diameter $D_f = f(\beta)$ for a OFHC copper and a 4340 steel. The initial dimensions of the projectile are $L_i = 37.97mm$ and $D_i = 7.595mm$ the speed at the impact is $V = 183m/s$ for the 4340 steel and $V = 182m/s$ for the OFHC copper. Results of this study are reported on figure 1 and 2.

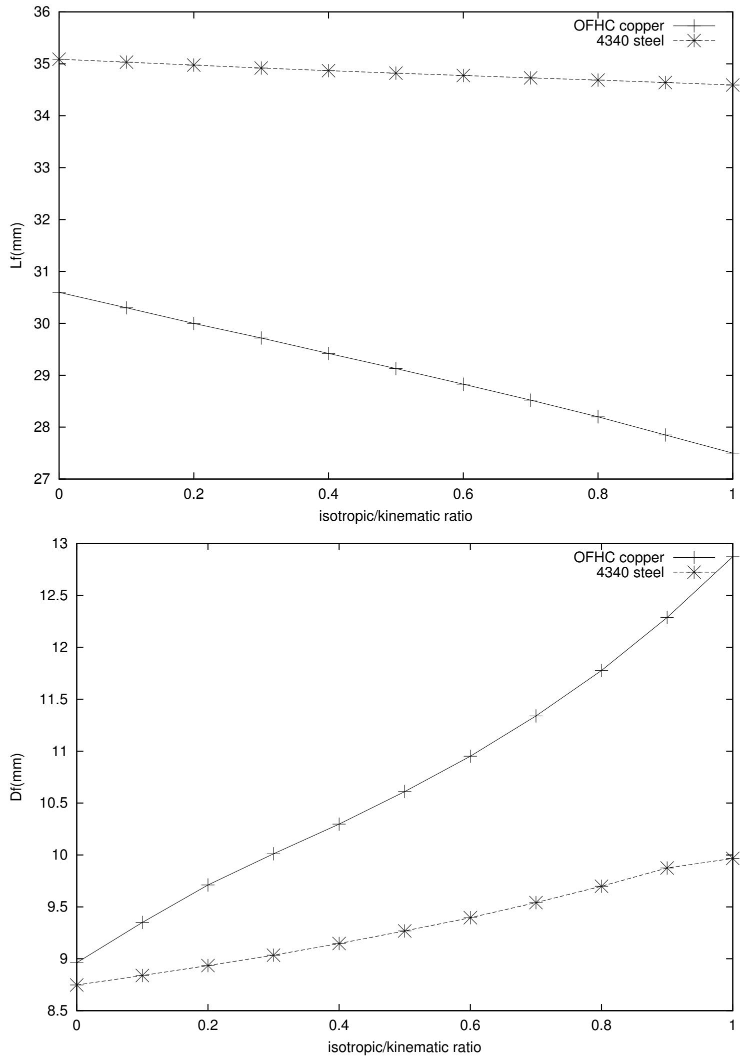

Figure 1: Influence of the β coefficient on final length and mushroom diameter

From those figures, we can see first that the final geometry of the OFHC have strong dependence to the β coefficient, particularly, final mushroom diameter varies in a range of 43% while β varies from 0 to 1. The second remark concerns the fact that an increase of the β coefficient, ie a more "isotropic" hardening, increase the final diameter and decrease the final length (a sort of softening of the material). Remembering that this has been done with all 5 Johnson-Cook flow law parameters constants, this study shows that an accurate identification procedure must take into account the 5 classic Johnson-Cook parameters and the β value. This 6 parameters identification procedure is presented in the next paragraph.

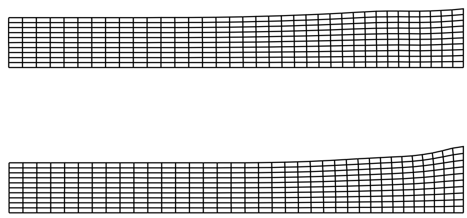

Figure 2: Influence of the β coefficient for the 4340 steel (above $\beta = 0$, below $\beta = 1$)

IDENTIFICATION OF MATERIAL PARAMETERS

The identification procedure

First we define the distance between computed and measured results by :

$$d = \sqrt{\sum_{i=1}^{r} \left(\frac{r_i - r_i^{exp}}{r_i^{exp}} \right)^2} \tag{8}$$

where the r_i constitute the set (r terms) of the responses chosen by the experimenter.
The procedure begins with the initial values proposed for the n parameters p_i, i .e the baseline point $p^0 = [p_1^0, ..., p_n^0]$, for which we calculate the distance d_0. For accounting to the various magnitudes of the parameters, a complete polynomial, dimensionless form is assumed for the function $\frac{d-d_0}{d_0}$, using variables $x_i = \frac{p_i - p_i^0}{p_i^0}$. Then the procedure can be decomposed into 4 steps :

- step 1 : a random distribution of q values of the parameters between specified bounds is performed, a Python script is automatically generated, then executing q runs by Abaqus/Design, extracting results and determining the set of computed responses $r_i (i = 1, ..., q)$. q depends on the number of parameters and on the degree of the polynome
- step 2 : by solving a linear q by q system, the coefficients α_i of the polynome can be determined (we use a C++ module developed in the laboratory)
- step 3 : an optimizer algorithm is used to minimize the function $\frac{d}{d_0}$ between fixed bounds
- step 4 : after an actualization, a single new run of Abaqus is automatically performed, then the step 2 to 4 are repeated, until the distance is less than a "noise distance" computed from the measurement precisions of the responses

Many versions for the step 4 (the core of the procedure) are available. For instance, the baseline point for the next iteration is built with the parameters issued from the previous iteration, so d_0 is the variable quantity; in an other version, d_0 is constant, but the "farthest" set of parameters is discarded with the new one... In all cases, note than only one Abaqus run is needed for each iteration.
A particular software has been developed to make the process automatic; it associates C, Python (Oriented Object Interpreted Language), the Abaqus command language and Fortran by the mean of "user subroutine". We use it to identify High Speed constitutive elasto-viscoplastic laws (associated with impact tests), surfacic friction laws (associated with quick extrusion tests), and damage laws (associated with shear tests), in the field of cutting of materials.

Application to the 4340 steel

We used our procedure to identify the 5 Johnson-Cook parameters and the β value for a 4340 steel. Experimental results $L_f = 34.7mm$, $D_f = 9.8mm$ for an initial speed $V = 183m/s$ were taken from literature [4]. Table 1 show the 5 identified parameters for an isotropic and a kinematic hardening and the 6 identified parameters for an isotropic/kinematic hardening law.

Table 1: Identified parameters for the 4340 steel

A (MPa)	B (MPa)	C	n	m	β	L_f (mm)	D_f (mm)	d (%)
829	483	0.2523	0.0138	1.0764	iso.	34.57	9.97	1.73
644	500	0.2851	0.0142	1.0228	kin.	34.64	8.94	8.77
810	507	0.2412	0.0154	1.0316	0.85	34.69	9.77	0.32

From this table, we can see that the isotropic hardening gives a good agreement of the final diameter while the kinematic hardening gives better results for the final length. The use of the norm defined by (8) and a combined isotropic/kinematic hardening law gives a very good agreement of the numerical and experimental results for both final diameter and length. We can also verify that the A coefficient varies in the same way as the β coefficient.

CONCLUSION

This study has shown the importance of the hardening law used in the FE computation for a discrete identification procedure. Concerning the contact law identification procedure, the same operations may be performed. We just have to change the experimental test from the direct Taylor impact to a conical high speed extrusion test, and of course the identified law.

References